

# HIGH RATE LOADING OF MATERIALS AND STRUCTURES SIMULATION ON THE BASE OF MOVABLE CELLULAR AUTOMATON METHOD

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## ABSTRACT

In the present paper the modified MCA method is proposed for simulation of high rate loading processes. Firstly, some test computations are presented. Then the problem of steel plate penetration by long tungsten alloy rod under normal impact is considered. The influence of impact velocity and plate thickness on penetration process pattern is studied. The dependences of final velocity and rod length as functions of impact velocity and plate thickness are presented. The calculated curves and overall behavior of simulated system demonstrated sufficient compliance with the experimental data, which proves that the MCA method is now applicable to a wider range of problems.

## 1 INTRODUCTION

Computer simulation of high-rate deformation processes provides the invaluable possibility to have the insight of process dynamics with unlimited temporal resolution, which is impossible in the experiment; it considerably reduces the number of experiments as well as expenses allowing the extrapolation of experimental data. The existing computational codes based on finite element or finite difference methods give the results (for example, Johnson [1], Summers [2]) that match reasonably well the experimental data. However, these methods cannot describe all aspects of high rate deformation process in the whole velocity range equally well and there is a demand for new computer models with different combinations of their weak and strong points. During the last decade the most promised are considered to be discrete approaches like smooth particle hydrodynamics (Johnson [3]), generalized particle algorithm (Johnson [4]), etc.

One of the modern discrete methods for modeling mechanical behavior of materials is the Movable Cellular Automaton method (MCA). It has proved itself to be a very effective and prominent instrument for simulation of materials and structures under rather small elastoplastic deformations (Psakhie [5,6,7]). The main advantage of the MCA method is its ability to naturally describe fracture of material up to its fragmentation. However, computational study of high-rate processes such as shock loading, high-velocity impact, etc. has not been available until recent time. Simulation of such phenomena has resulted in incorrect behavior due to big automata overlaps. In order to overcome this limitation and expand the velocity range available for simulation by this method certain modifications are proposed to be introduced. The most important of them is the modification of automata response function assuming that the bulk modulus of the material under compression grows with the increase of bulk strain. In the paper it is suggested to introduce non-linearity in the MCA method by modification of the elastic modulus, making it a function of automata overlap, which corresponds to growth of force response with increase of bulk strain.

It is no doubt that for more accurate simulation of high rate deformations the introduction of nonlinear response function is not the only improvement required. For instance, we also have to

account for direct dependence of stress on strain rate and reconsider the viscous forces computation, which have been developed for quasistatic deformations. Note, that the improvement discussed is very important also for simulation of quasistatic processes where big bulk compression occurs, for example, the deformation under constrained boundary conditions similar to the processes in geological media.

## 2 MODIFICATION OF THE METHOD FOR SIMULATION OF HIGH-RATE PROCESSES

Within the framework of the MCA method equations of motion for automata (see Psakhie [5,6]) can be written as

$$\begin{cases} m^i \frac{d^2 \mathbf{R}^i}{dt^2} = \mathbf{F}_\Omega^i + \sum_j \mathbf{F}^{ij} \\ \hat{J}^i \frac{d^2 \bar{\theta}^i}{dt^2} = \sum_j \mathbf{K}^{ij} \end{cases}, \quad (1)$$

where  $m^i$  is the mass of automaton  $i$ ,  $\hat{J}^i$  is its moment of inertia,  $\mathbf{R}^i$  is the position,  $\bar{\theta}^i$  is the angle of the relative rotation,  $\mathbf{F}^{ij} = \mathbf{p}^{ij} + \boldsymbol{\tau}^{ij}$ ,  $\mathbf{p}^{ij}$  is the central component of the interaction force in the pair  $ij$ ,  $\boldsymbol{\tau}^{ij}$  is the tangential component of this force,  $\mathbf{F}_\Omega^i$  is the volume-dependent force acting on the automaton  $i$ ,  $\mathbf{K}^{ij} = q^{ij} (\mathbf{n}^{ij} \times \boldsymbol{\tau}^{ij})$ ,  $q^{ij}$  is the distance from the center of the automaton  $i$  to the point of its contact with the neighbor  $j$ ,  $\mathbf{n}^{ij}$  is the unit vector defined as  $\mathbf{n}^{ij} = (\mathbf{R}^j - \mathbf{R}^i) / (q^{ij} + q^{ji})$ .

This formulation explicitly accounts for the response caused by the volumetric changes. The volume-dependent force  $\mathbf{F}_\Omega^i$  acts due to the pressure from all the neighboring automata. The automaton pressure is determined by the change in the automation volume. In the simplest (linear) case this dependence takes the form

$$P^j = K^j \varepsilon^j \quad (2)$$

where  $\varepsilon^j = (\Omega^j - \Omega_0^j) / \Omega_0^j$  is the bulk deformation,  $\Omega_0^j$  is the initial (equilibrium) volume of automaton  $j$ ;  $\Omega^j$  is the current volume and  $K^j$  is the bulk modulus. The change in the automaton volume in a time  $\Delta t$  corresponds to computation based on the corresponding changes in the distances  $\Delta q^{jk}$  from the center of this automaton to the points of its contact with neighbors, i.e.

$$\Delta \Omega^j = \sum_{k=0}^{N^j} \Delta q^{jk} S^{jk}, \quad (3)$$

where  $N^j$  is the number of neighbors of the automaton  $j$ ,  $S^{jk}$  is its contact square with the neighbor  $k$ .

As it was mentioned above in the case of big compression the pressure in eqn (2) must grow much faster than linear dependence of bulk strain. It is well known that the experimental data on thermodynamical properties of materials in wide range of pressure is described by the shock adiabat which is defined by the linear relation (Gust [8])

$$D = c_0 + bU, \quad (4)$$

where  $D$  is the shock-wave velocity, and  $U$  is the mass velocity of the substance beyond the shock-wave front. Material constants  $c_0$  and  $b$  has been measured for a wide range of materials and can be easily found in literature.

From eqn (4) and from the Hugoniot relations for conservation of mass and momentum

$$\varepsilon = -U/D, \quad P = \rho DU, \quad (5)$$

the following relation can be derived

$$P = \rho c_0^2 \frac{\varepsilon}{(1+b\varepsilon)^2} = K \frac{\varepsilon}{(1+b\varepsilon)^2}. \quad (6)$$

It is the simplest non-linear dependence of pressure on bulk strain. To use it in the MCA method it is natural to redefine the bulk elastic modulus  $K$  in eqn (2) as

$$K' = \frac{dP}{d\varepsilon} = K \frac{1-b\varepsilon}{(1+b\varepsilon)^3}. \quad (7)$$

Thus, the introduced non-linear equation of state requires only one additional parameter  $b$  that determines the inclination of  $D-U$  diagram. It can be found from the analysis of experimental shock adiabats or directly from literature.

### 3 COMPUTATION RESULTS

The first step in testing and verification of the new modification of the MCA computational technique is simulation of one-dimensional chain of automata, which allows comparing with analytic solutions. The one-dimensional chain is a very good illustration that linear model is suitable only for very small deformations.

We considered compression of the one-dimensional chain of automata in the following statement. The right end automaton of the chain was fixed and the left one was moved to the right with the constant velocity  $u$  varying from 1 to 500 m/s. The force acting on the left end was measured as a function of chain deformation. The maximum deformation applied was up to 0.75 of initial length. Simulation results show that the loading with the constant velocity produces compression pulse that traverses the chain to the right and then, after reflection, to the left. Reflection from the left boundary is associated with stress increase on the stress-strain curve followed by horizontal segment associated with the pulse trip between the boundaries. As a result the loading curve is a staircase one (Fig. 1). If the automaton response is pure elastic then delay between pulses is constant regardless the current value of chain length. This means that velocity of

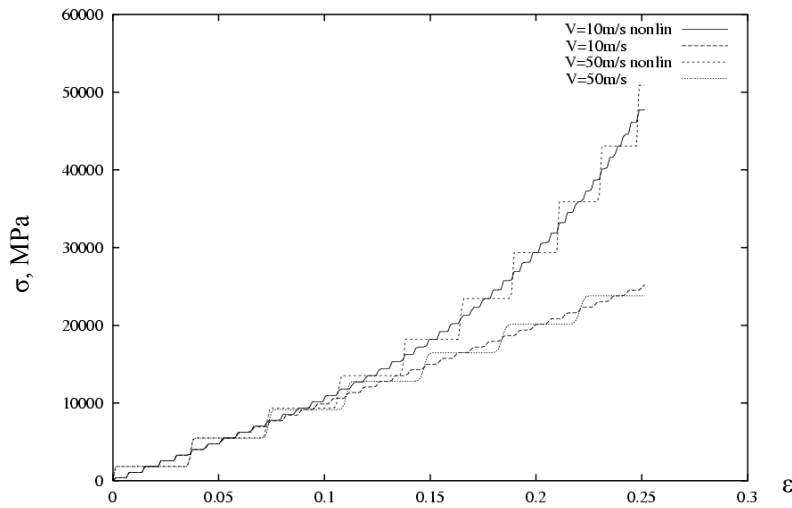


Figure 1: Reaction of linear and non-linear elastic automata chains under different loading rates.

the pulse (sound velocity) is decreasing proportionally to the chain length, confronting the common sense. Explanation can be given by the fact that in the considered discrete system of automata the distance for pulse propagation is measured in the units of number of automata not in spatial ones. In other words the density of the chain is not changed under such conditions. Hence, for the system with linear interaction even big compression does not make any difference in sound velocity. On the contrary, if automata interaction is non-linear like (6), the stairs in Fig. 1 become shorter and higher as the compression grows. It means that the sound velocity  $c$  is growing according to formula  $\sqrt{E/\rho}$ , where  $E$  is the current elastic modulus,  $\rho$  is the density. The height of the stairs can be roughly estimated with aid of the formula

$$P = \rho c u. \quad (8)$$

Hence, the increasing of  $c$  leads to the increase of the stairs height. The eqn (8) determines the pressure in compression of pulse derived in the acoustic approximation.

Let us consider how the behavior of one-dimensional chain under loading is changed for plastically deformed material. For elastic material the value of pressure is directly proportional to loading velocity according to (8). For plastic material situation is different: pressure grows faster than  $u$ . From the loading diagram it can be seen that after the transient period consisting of the first peak, drop, and then growth the constant plateau follows denoting that the plastic wave has been formed. The first peak can be connected with domination of viscous force over the elastic one. In Fig. 2 one can see the particle velocity profile for such kind of material, it illustrates propagation of the elastic precursor spreading faster and the plastic wave spreading slower.

Simulation of compression tests in two-dimensional statement with the same materials and loading rates showed qualitatively the same results. To avoid specimen inflation in the transversal direction it was required to introduce constrained or periodical boundary conditions. The influence of constrained boundaries resulted in negligible changes in the stress-strain diagrams, but the basic behavior is the same with the compression of one-dimensional chain.

As an illustration of applicability of the improved MCA method let us consider the results of simulation of steel plate penetration by long tungsten rod under normal impact in two-dimensional statement. Plate thickness was varied from 15 up to 40 mm with step of 5 mm. The plate width was limited to 150 mm. To imitate wider plates the specific constrained boundary conditions are applied to the left and right sides of the plate. The conditions imply damping of horizontal component of velocity with certain coefficient assigned to 0.1 in our case. The dimensions of impacting rod were 10x100 mm, the impact velocity was 1520 m/s. The task geometry and loading conditions were taken to correspond to calculations described by Fomin [9].

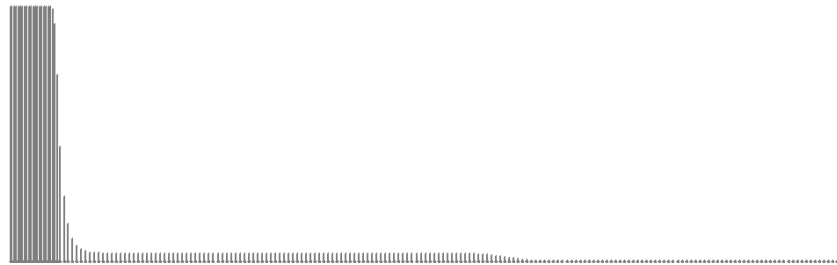


Figure 2: Profile of wave front (velocity distribution) in elastoplastic automata chain presenting faster elastic impulse and slower plastic wave.

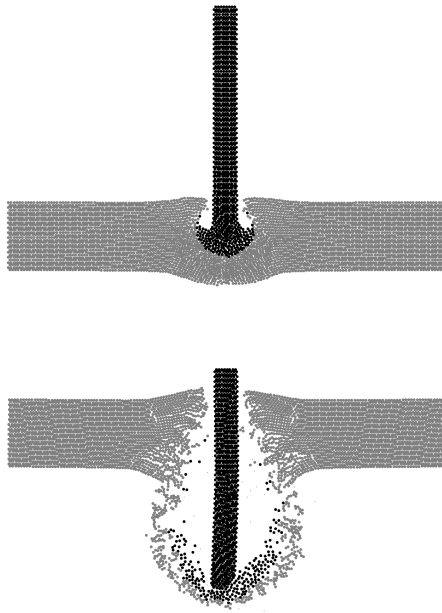


Figure 3: Structures of the penetrating rod and the target plate at 20 and 60  $\mu\text{s}$ .

Thickness, [mm]	$\Delta U/U_0$	$\Delta l/l_0$
15	0.04	0.20
20	0.05	0.21
25	0.06	0.23
	0.05*	0.24*
	0.02**	0.19**
	0.026***	0.29***
30	0.08	0.26
35	0.10	0.30
40	0.12	0.39

Table 1: Relative decrease of the velocity ( $\Delta U/U_0$ ) and the length ( $\Delta l/l_0$ ) of the rod after penetration for different target plate thicknesses. For the thickness of 25 mm one can compare our results with data of other authors: \* – experimental data from Fomin [9], \*\* – computer simulation results by Johnson [1], \*\*\* – computer simulation results from Fomin [9].

Fig. 3 shows an intermediate and the final states of the simulated structure. This picture exhibits formation of lips at the beginning and at the end of the penetration channel. One can see that stochastic distribution of strength properties of automata in the plate leads to bending of the rod due to its instability that can also occur in practice. Table 1 presents data on relative decrease of the velocity and the length of the rod after penetration. Fig. 5 represents time history of the rod

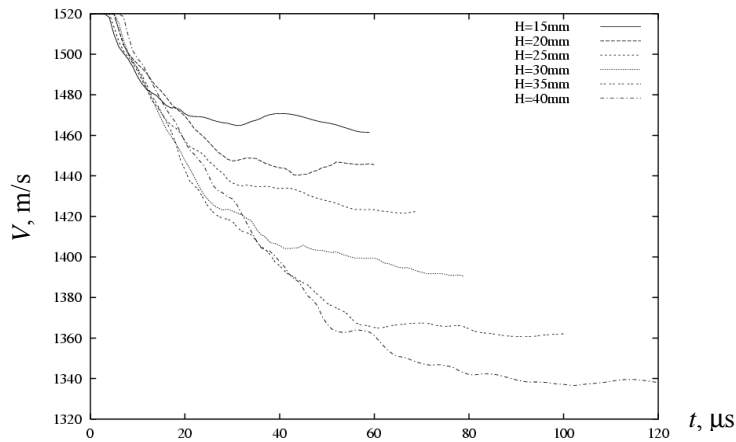


Figure 5: Velocity of the penetrating rod versus time for different plate thicknesses.

velocity for different thickness of the target during the penetration process. According to this diagram, the final velocity decreases with plate depth faster than linear.

#### 4 CONCLUSION

The influence of the new nonlinear term in automaton response function on the behavior of simulated samples under various loading conditions is studied on the number of test problems. Calculations of one-dimensional chain and two-dimensional compression test have shown that the response of simulated systems became much more realistic for big deformation. The introduced modification allows to describe hardness growth and growth of sound velocity in compressed medium.

Comparing with analytical estimations made in the frame of the acoustic approximation and with the experimental data shows that the obtained results have sufficient compliance with them.

The modified MCA method is applied to simulation of finite thickness steel slab penetration by long heavy tungsten alloy rod. The influence of such parameters as impact velocities, slab thickness on penetration process pattern is investigated. The dependences of final velocity and rod's length as functions of impact velocity and slab thickness are calculated. The calculated curves and overall behavior of simulated system has demonstrated sufficient compliance with the experimental data

The strengthened response of automata helps to avoid too big automata overlap that makes it possible to simulate high-rate deformation phenomena. The increase of material elastic modulus with volume strain is also very important for description of processes where big static volume compression takes place, for example, deformation of geological media.

#### REFERENCES

1. Johnson, G.R., Stryk, R.A., Holmquist, T.R., et al., *Int. J. Impact Engng.*, **10**, pp. 281, (1990)
2. Summers, R.M., Peery, J.S., Wong, M.K., et al., *Int. J. Impact Engng.*, **20**, pp. 779, (1997)
3. Johnson, G.R., Stryk, R.A., Beissel, S.R., *Comput. Meth. Appl. Mech. Engng.*, **139**, pp. 347, (1996)
4. Johnson, G.R., Beissel, S.R., Stryk, R.A., *Comput. Mech.*, **25**, pp. 245, (2000)
5. Psakhie, S. G., Ostermeyer, G., Dmitriev, A. I., et al., *Comput. Mater. Sci.*, **16**, No. 4, pp. 333–344, (1999)
6. Psakhie, S. G., Ostermeyer, G., Dmitriev, A. I., et al., *Phys. Mesomechanics*, **3**, No. 2, pp. 5–12, (2000)
7. Psakhie, S. G., Horie, Y., Ostermeyer, G. P., et al., *Theor. App. Fract. Mech.*, **37**, pp. 311–334, (2001)
8. Gust, W.H., *J. Appl. Phys.*, **53**, No.5, pp. 3566, (1982)
9. Fomin, V.M., Gulidov, A.I., Sapozhnikov, G.A., et al., *Vysokoskorostnoe vzaimodeistvie tel*, Izdatelstvo SO RAN, Novosibirsk, 600 p., (1999) (In Russian)