FRACTAL GEOMETRY AS AN INCIPIENT THEORY OF ROUGHNESS

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ABSTRACT

Roughness is, among human sensations, just as fundamental as color or pitch, or as heaviness or hotness. But its study had remained in a more primitive state, by far. The reason was that both geometry and science were first drawn to smooth shapes. Thus. color and pitch came to be measured in cycles per seconds, that is, were reduced to sinusoids, in other words to uniform motions around a circle - the epitome of a smooth shape. A study of roughness had necessarily to wait until specific mathematical tools had been discovered and, much later, suitably interpreted.

Fractal geometry began when I reinterpreted the flight from nature that had led mathematicians to conceive of notions like the Holder exponent, the Cantor set, or the Hausdorff dimension. They boasted of these notions being "monstrous" but in fact I turned them over into everyday tools of science. I also added further tools that - taken together - made roughness quantitatively measurable for the first time.

Acquiring a quantitative measure is the step that moves a field into maturity. And this move instantly led to a striking conjecture. In 1984, "Nature" published an article I wrote with D. Passoja and Paullay on metal fractures. We found that the traditional measures of their roughness range all over. To the contrary, their fractal roughness varies very little not only between samples but also between materials. Last time I checked the "universality" had been extended but not explained. The new intrinsic measure created a major intellectual mystery.

The first major new tool that I added to those contributed by the likes of Holder, Cantor, and Hausdorff was multifractality, for both measures and functions. I was motivated by the urge to model the intermittency of turbulence but my first full paper (in 1972) also noted that the same techniques ought to apply to the intermittency in the variation of financial prices. An ancient adage claimed that the City of London is as unpredictable as the weather. I found unexpectedly quantitative truth to this adage by showing that both phenomena can be tackled with essentially the same tools.

Roughness is everywhere therefore fractal geometry has little fear of running out of problems. This address will sketch the fractal geometry of roughness and explore some new developments relevant to this Congress.